Sub Code- 216

HISSAN KASKI-Grade XII

Pre- Board Examination – 2071

Compulsory Mathematics

Programme:Science **Full Marks: 100**

Time: 3hrs **Pass Marks: 35**

Shift : Morning

Candidates are required to give their answers in their own words as far as practicable. the figures in the margin indicate full marks

Attempt all the questions of group A and group B or C.

Group 'A'

1. a. In an examination paper containing 12 questions, a student has to answer 10 questions. If two questions are made compulsory, in how many ways can be choose 10 in all? [2]

   b. Find the term independent of x in the expansion of $\left(\frac{3x^2}{2}-\frac{1}{3x}\right)^9$. [2]

   c. If (G,*) is a group and $a,b \in G$, then prove that $(a*b)^{-1} = b^{-1}*a^{-1}$. [2]

2. a. Find the equation of parabola with vertex at (-1,2) and directrix x=2. [2]

   b. Find the equation of plane through the point (3,-4,5) and parallel to the plane 3x-4y+5z = 7 [2]

   c. If $3\vec{i}+\vec{j}-\vec{k}$ *and* $\lambda\vec{i}-4\vec{j}+4\vec{k}$ are collinear vectors, find the value of λ. [2]

3. a. Find the derivative of $x^{\cosh^2\frac{x}{a}}$. [2]

   b. Give the geometrical interpretation of scalar product of two vectors. [2]

   c. Evaluate the integral $\int\frac{dx}{\sqrt{(x-\alpha)(x-\beta)}};\beta>\alpha$ [2]

4. a. Solve the differential equation $y' = e^{x-y}+x^2.e^{-y}$ [2]

   b. From a group of 10 items, $\sum X = 452, \sum X^2 = 24270$ and mode =43.7. Find the pearsonian coefficient of skeweness. [2]

   c. A card is drawn from a well shuffled deek of 52 cards. What is the probability that it is king or a club? [2]

5. a. In how many ways can the letters of the word 'COMPUTER' be arranged so that

      i. all the vowels are always together.

      ii. the vowels may occupy only odd position? [4]

   b. Show that the set G = {1,-1, i, -i }, where i is an imaginary unit, forms a group under multiplication operation.

   OR

   If (G,*) is a group with binary operation '*' and if a,b $\in$ G, then the equation a*x =b and y*a = b have unique solution x and y in G. [4]

6. a. Find the equation of common tangent of parabolas $y^2=4ax$ and $x^2= 4by$.

   OR

   Find the eccentricity, co-ordinates of vertices and foci, and the length of latus rectum of ellipse $x^2+4y^2-4x+24y+24= 0$. [4]

   b. Prove that the lines whose direction cosines are given by the relation al+bm+cn = 0 and fmn+gnl+hlm = 0. Are perpendicular if $\frac{f}{a}+\frac{g}{b}+\frac{h}{c}=0$. [4]

7. a. Evaluate the integral $\int\frac{dx}{a+b\cos x}$, *where* $a>b$. [4]

   b. Solve the homogeneous differential equation $2\frac{dy}{dx}=\frac{y}{x}+\frac{y^2}{x^2}$

   OR

   Solve the linear differential equation $\cos^2 x.\frac{dy}{dx}+y=1$. [4]

8. a. From the data given below find:

   i. line of regression of the marks in Mathematics on the mark in English.

ii. the most likely marks in Mathematics when the marks in English is 30.

Marks in English	25	28	35	32	31	36	29	38	34	32
Marks in Maths	43	46	49	41	36	32	31	30	33	39

[4]

b. Five men in a group of 20 are graduates. If 3 men are choosen out of 20 at random what is the probability that at least one being graduates? [4]

9. Show that $1+\frac{1+2}{2!}+\frac{1+2+3}{3!}+\frac{1+2+3+4}{4!}+......to\infty=\frac{3e}{2}$ [6]

10. Using vector method prove that
Sin(A-B) = SinA.CosB – CosA Sin B. [6]

11. Find from first principles, the derivative of $\ln(\sin^{-1}x)$

OR

State Rolle's theorem and interpret it geometrically. Verify Rolle's theorem for the function $f(x)=e^{x}.\cos x\, on[-\frac{\pi}{2},\frac{\pi}{2}]$

[6]

Group 'B'

12. a. If a force p be resolved into two forces making angles 45^0 and 15^0 with its direction , show that the later force is $\frac{\sqrt{6}}{3}p$.

[2]

b. A straight uniform rod is 3m long. When a load of 10N is placed at one end, it balances about a point 25cm from that end. Find the weight of the rod. [2]

c. A ball thrown up vertically upwards returns to the thrower after 6 seconds. Find the velocity with which it was thrown up.

[2]

13. a. A body of weight 65N is suspended by two strings of lengths 5m and 12m attached to two points in the same horizontal line whose distance apart is 13m. Find the tension on the strings.

OR

Find the resultant of two forces P and Q acting at a point when the angle between them is α. [4]

b. If a, b, c are the spaces described by a particle during p^{th},q^{th} and r^{th} seconds of its motion respectively prove that a(q-r) +b(r-p) + c(p-q) =0. [4]

14. A particle is projected with a velocity u. If the greatest height attained by the particle is H, prove that the range R on the horizontal plane through the point of projection is $R=4\sqrt{H\left(\frac{u^2}{2g}-H\right)}$

OR

A bullet of mass 200gm is fired into a target with a velocity of 500m/s. If the mass of the target is 4.8kg, and is free to move, find the loss of kinetic energy by the impact . [6]

15. Forces equal to 3p, 4p and 5p act along the sides AB, BC and CA of an equilateral triangle ABC. Find the magnitude, direction and the line of action of the resultant. [6]

Group 'C'

16. a. Shade the feasible region determined by the inequalities $x+2y\le 7, x-y\le 4, \quad x,y\ge 0.$ [2]

b. Convert the octal numeral $(2630)_8$ into hexadecimal form [2]

c. Using trapezoidal rule, evaluate $\int_0^{\frac{\pi}{2}}\sqrt{\sin x}dx, \quad n=2$ [2]

17. a. Solve the system of equation

$x_1-2x_2+3x_3=10$

$2x_1+3x_2-2x_3=1$

$-x_1-2x_2+4x_3=13,$

using Gauss elimination method.

OR

Solve the following system of equation by Gauss Seidel method:
2x+y+4z=8, 3x+5y+2z=15, 2x+y+z= 5. [4]

b. Evaluate the approximate value of $\int_0^1 \frac{1}{1+x^2}dx$ with n=4 using simpson's $\frac{1}{3}$ rule. Also calculate error and error bound. Finally verify the result. [4]

18. Using simplex method maximize :
Z = 2x + 3y subject to the constrains,
$2x + y \leq 14$
$x + 2y \leq 10$ [6]
$x, y \geq 0.$

19. Using the method of successive bisection, find the square root of 3 with two places of decimal (1,2).

OR

Find the square root of 612 correct to three places of decimals using Newton- Raphson method. [6]

*** Thank You***