

HISSAN KASKI-Grade XII

Pre- Board Examination – 2071

Compulsory Mathematics

Programme: Science

Full Marks: 100

Time: 3hrs

Pass Marks: 35

Shift : Day

Candidates are required to give their answers in their own words as far as practicable. the figures in the margin indicate full marks

Group 'A'

1. a. In how many ways can eight people be seated around a table if 3 people insist on sitting next to each other? 2
- b. Find the coefficient of x^5 in the expansion of $\left(x + \frac{1}{2x}\right)^7$ 2
- c. For the set $S = \{-1, 1\}$, show that the operation represents the usual multiplication is a Binary operation. 2
2. a. Find the eccentricity of $9x^2 + 5y^2 - 30y = 0$ 2
- b. The projection of line on the axes are 1, 2, 2, find the direction cosines of the line. 2
- c. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ then prove that the vectors \vec{a} and \vec{b} are perpendicular to each other. 2
3. a. Using L Hospital Rule, evaluate: 2

$$\lim_{x \rightarrow 0} \frac{x - \sin x}{\tan^3 x}$$
- b. Evaluate: $\int \frac{dx}{e^x + e^{-x}}$ 2
- c. Find the area of a parallelogram whose adjacent sides are $\vec{i} + \vec{j}$ + \vec{k} and $2\vec{i} - 3\vec{j} - \vec{k}$. 2
4. a. Solve: 2

$$\frac{dy}{dx} = e^{x-y} + x^2 \cdot e^{-y}$$

- b. Find the standard deviation of the data 2, 3, 5, 7, 11. 2
- c. Given $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cup B) = 0.56$. Are A and B independent? 2
5. a. In how many different words can be formed of the letters of the word "MALENKOV" so that, 4
 - i. Words in which no two vowels are together.
 - ii. Words in which vowels being always together.
- b. Given that the algebraic structure $(G, *)$ with $G = \{1, \omega, \omega^2\}$ where ω represents the imaginary cube root of unity and $*$ stands for the binary operation of multiplication. Show that $(G, *)$ is a group. 4

'OR'

Show that the set of integers Z forms a group under the operation of addition.

6. a. Find the condition that the line $y = mx + c$ may be the tangent to circle $x^2 + y^2 = a^2$. 4

'OR'

Show that $9x^2 + 4y^2 - 18x - 16y - 11 = 0$ represents the equation of an ellipse. Find its centre, focus, eccentricity and directrix.

- b. Find the equation of the plane through the point (2, -3, 1) and perpendicular to the line joining the points (3, 4, -1) and (2, -1, 5). 4
7. a. Evaluate: 4

$$\int \frac{dx}{3 \sin x + 4 \cos x}$$
- b. Solve: 4

$$(3xy + y^2) dx + (x^2 + xy) dy = 0$$

'OR'

$\cos^2 x \frac{dy}{dx} + y = \tan x$

8. a. Calculate the Karl's Pearson's correlation coefficient between height and weight. 4

Height (cm)	170	172	175	171	173
Weight (kg)	73	72	74	70	71

- b. A dice is thrown 3 times getting an odd number is a success. Find the probability of getting, 4
- i. 3 successes ii. no success

9. If $(1 + x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n$, prove that,

$$C_0C_n + C_1C_{n-1} + \dots + C_nC_0 = \frac{2n!}{(n!)^2} \quad 6$$

10. Define the vector product of two vectors then prove by vector method that,

$$\sin(A + B) = \sin A \cos B + \cos A \sin B \quad 6$$

11. State mean value theorem. Interpret it geometrically. Verify mean value theorem for the function

$$f(x) = x(x - 1)(x - 2) \text{ in } \left[0, \frac{1}{2}\right] \quad 6$$

'OR'

Find from first principle, the derivative of Insecx .

Group 'B'

12. a. Two forces acting at an angle of 45° have a resultant equal to $\sqrt{10}N$. If one of the force be $\sqrt{2}N$. Find the other force. 2
- b. A straight uniform rod is 3 m long. When a load of 5 N is placed at one end, it balances about a point 25 cm from that end. Find the weight of the rod. 2
- c. A body is projected vertically with a velocity 9.8 ms^{-1} , how long it takes to return to the point of projection. 2
13. a. Two forces P and Q acting parallel to the length and base of an inclined plane respectively would each of them singly support a weight W on the plane.
- Prove that, $\frac{1}{P^2} - \frac{1}{Q^2} = \frac{1}{W^2}$ 4

'OR'

State and prove triangle of forces.

- b. If a, b, c be the space described by a particle during the p^{th} , q^{th} and r^{th} seconds of its motion respectively prove that,
- $$a(q - r) + b(r - p) + c(p - q) = 0 \quad 4$$
14. From a point on the ground a distance x from the foot of a vertical wall, a ball is thrown at an angle of 45° which just clears the top

of the wall and afterwards strike the ground at a distance y on the other side, prove that the height of the wall is $\frac{xy}{x + y}$. 6

'OR'

State the principle of conservation of energy. Prove the sum of K.E. and P.E. of a freely falling body at any instant is constant.

15. State and prove Varignon's theorem for the forces meeting at a point. 6

Group 'C'

16. a. Shade the feasible region of $2x + y \leq 40$, $x + 2y \leq 50$, $x > 0$, $y > 0$. 2
- b. Convert the octal numeral 3733_8 into decimal number. 2
- c. Using trapezoidal rule to evaluate
- $$\int_1^2 \frac{1}{x} dx, n = 4 \quad 2$$

17. a. Solve the following system of equations using Gaussian elimination method. 4
- $$\begin{aligned} x - 2y + 3z &= 2 \\ 2x - 3y + z &= 1 \\ 3x - y + 2z &= 9 \end{aligned}$$

'OR'

Solve by Gauss Seidal method,

$$3x + y - z = 2, 2x - 5y + z = 20, x - 3y - 8z = 3$$

- b. Using Simpson's $\frac{1}{3}$ rule, evaluate $\int_0^1 \frac{dx}{1+x^2}$, $n = 4$ 4

18. Using simplex method,
- Max. $Z = 7x_1 + 5x_2$ subject to
- $$\begin{aligned} 2x_1 + 3x_2 &\leq 13 \\ 3x_1 + 2x_2 &\leq 12 \\ x_1, x_2 &\geq 0 \end{aligned} \quad 6$$
19. Using successive bisection, find the root of $x^3 - 4x + 1 = 0$ lying between 1 and 2 correct to 2 dp. 6

'OR'

Using Newton Raphson method, find the positive root of $x^3 - 2x - 5 = 0$ in (2, 3) correct to 3dp. 6

Best of Luck